Geometry

Category 1 – Geometric Structure

HISTORICAL DEVELOPMENT

Euclid published *The Elements* in 300 BC describing Euclidean geometry. His listing of assumptions, definitions, and logically proven statements provided a template for other geometries that describe real world and abstract phenomena.

axioms or postulates: statements that are assumed to be true without proof

Example: Segment Addition Postulate: If point B lies on $\overline{AC}$, then $AB + BC = AC$

theorem: statement that has been proven true using only axioms or postulates, definitions, other theorems, and logic **Example:** Pythagorean Theorem

geometric system: consistent axiomatic system useful for describing spatial (location-based) properties and relationships (real world or abstract)

Examples: Architects use Euclidean geometry to design buildings, which contain many geometric shapes. Ship captains and airplane pilots use spherical geometry to find the shortest path from one point to another.

EUCLIDEAN AND NON-EUCLIDEAN GEOMETRIES

Euclidean geometry: consists today of many axioms or postulates, including:
1. Through any two points there is exactly one line.
2. A straight line can extend indefinitely from a given point.
3. A circle can be defined using a given point as its center and a radius.
4. Things which are equal to the same thing are **congruent** (≅) to each other.
5. *Parallel Postulate:* Two straight lines will cross at exactly one point if the of the interior angles on a side generated by a transversal is less than

Non-Euclidean geometries: geometries that do not adhere to the *Pa Postulate*, such as hyperbolic or elliptical (spherical) geometries

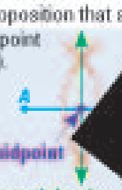
Geometry	Line	Plane	Angles
Euclidean	straight	flat	add to 180°
spherical	great circle	spherical surface	

GEOMETRIC CONSTRUCTIONS

geometric constructions: drawings made using a st

conjecture: unproven proposition that appears correct

Example 1: Find the midpoint of line segment AB ($\overline{AB}$).



2. Draw line between arc intersections with straightedge.
3. The midpoint is where line crosses $\overline{AB}$.

Example 2: W rela betw and

the (⊥) bisector of $\overline{AB}$.

MAKING AND VALIDATING CONJECTURES

Use axiomatic, coor formational approache conjectures.

Example 1: Mak about $\triangle ABC$ $B'C'$.

conjecture: $\triangle A$

validity: to be true). $\triangle ABC$ translated 4 units right and up is the same as $\triangle A'B'C'$.

	$A' = ($		
B	$B' =$		
$C =$		4	1

Ex ake a conjec relationship be the number of diameter dra cle and the nu of regions created. If one or unique diameters is d circle, the number of regions created is twice the number of diameters drawn.

lgebrai can used to test or prove co

	Description
ty	If $a =$ $a + c = b + c$.
property	If $a = b$, then $a - c = b - c$.
ication prope	If then $ac = bc$.
division p	b and $c \neq 0$, then $a \div c =$
reflexiv	any real number a, $a = a$.
symme	If $a = b$, then $b = a$.
transitive property	If $a = b$ and $b = c$, then $a = c$.
substitution property	If $a = b$, then a can be substit any equation or expression.
distributive property	$a(b + c) = ab + ac$

Example: given: $m\angle 3 = 90°$ conjecture: If $m\angle 1 = m\angle 2$, then $m\angle$
validity: True. By Triangle Sum of Angles Theorem $m\angle 1 + m\angle 2 +$
Given that $m\angle 3 = 90°$, then by substitution property $m\angle 1 + m\angle 2 +$
and then by subtraction property $m\angle 1 + m\angle 2 = 180° - 90° = 90$
$m\angle 1 = m\angle 2$, then by substitution property $m\angle 2 + m\angle 2 = 90°$, and by distributive property $2(m\angle 2) = 90°$, and therefore by division property $m\angle 2 = 45°$.

CONDITIONAL STATEMENTS AND REASONING

If a conditional statement is always t he contrapositive is always true.

Statement	Example tion of Statement's Validity
conditional (if p then q)	If $\angle A$ a ght angl then $m\angle A = m\angle B$. → TRUE
converse (if q then p)	If m then $\angle$ are right angles. → NO ARILY T
inverse (if not p then not q)	$\angle B$ are n gles, the $\neq m\angle B$ → SAR
contrapositive (if not q then not p)	and $\angle B$ angles → TR

If a biconditional statement (if a is tru se is a

Example: biconditional: If and only i t ang UE
converse: If and only if $\angle A$ is a right a → TRU

INDUCTIVE REASONING

inducti easoning: process of reasoning that exte idual example(s) or observati n(s) to sion; use to form con

e: 2, 12 a 32 are even numbers. c e: If a whole number en 2, then it is even.

DEDU NING

deductive r -step easoni ic) that applies known a new ements us axioms and theorems

Geomet	Pro Statement
plement **s ry angles:** t s that add such as with a ar pair)	If and $\angle B$ are supplementary same angle (or to congruent ngles), then $\angle A \cong \angle B$.
Con mplements **c tary angles:** two angl o 90°	If $\angle A$ and $\angle B$ are complementary to the same angle (or to congruent angles), then $\angle A \cong \angle B$.
ngles	...are congruent ($\angle ABC \cong \angle DBE$).
e Sum	The sum of the measures of the angles of any triangle is 180°.
Theorem	For any right triangle, $a^2 + b^2 = c^2$
nes a ected by a transversal, then:	
Cor g Angl	...are congruent ($\angle 1 \cong \angle 5$).
Consec e Inte ngles	...are supplementary ($m\angle 3 + m\angle 5 = 180°$).
Alternate Inter gles	...are congruent ($\angle 4 \cong \angle 5$).
nate Ext ngles	...are congruent ($\angle 2 \cong \angle 7$).

convers each parallel line theorem is also a theorem (proven true).

ample: Corresponding Angles Converse: If two lines are intersected by a transv uch that corresponding angles are congruent, then lines are parallel.

PRO AND COUNTER-EXAMPLES

pr of: written statements that show the deductive reasoning process that links information, definitions, axioms, and theorems to create certain conclusions

Example 1: 2-column proof
given: $\angle C$ is a right angle
prove: $\angle A$ and $\angle B$ are complementary

Statements	Reasons
1) $\angle C$ is a right angle	1) Given
2) $m\angle C = 90°$	2) Definition of right angle
3) $m\angle A + m\angle B + m\angle C = 180°$	3) Triangle Sum of Angles Theorem
4) $m\angle A + m\angle B + 90° = 180°$	4) Substitution property of equality
5) $m\angle A + m\angle B = 90°$	5) Subtraction property of equality
6) $\angle A$ and $\angle B$ are complementary	6) Definition of complementary angles

Example 2: paragraph proof
given: $\angle 1 \cong \angle 2$
prove: lines m and n are perpendicular (⊥)

You are given that $\angle 1 \cong \angle 2$, so by the definition of congruent angles $m\angle 1 = m\angle 2$.

= 180°,

o by (⊥).

$\overline{BD}$

ion of ruent ents

ly.

nter

Example: statement: If a polygon has a right angle, it is a triangle.
examples: counter example: Statement is disproved.