Algebra I EOC

DYNA NOTES™ — TOOLS FOR EXPLOSIVE LEARNING

Category 1 – Number and Algebraic Methods

11: AB SIMPLIFYING EXPRESSIONS WITH EXPONENTS AND RADICALS (CR)

base: value raised to the **power** shown by the **exponent** (superscript)

Example: In $3x^2y$, base x is raised to the second (2^{nd}) power (squared); if there is no exponent, the power is 1 (i.e., $3x^2y = 3^1x^2y^1$).

root: shown with **fractional exponent** or **radical sign**, $\sqrt{\ }$ (e.g., $x^{\frac{1}{2}} = \sqrt{x}$).

Law of Exponents	Description	Examples
$a^n = 1 \cdot a \cdot a \cdot a \ldots$ (n times)	powers of positive integers → repeated multiplication by base	$2^3b^2 = (1)(2)(2)(2)(b)(b) = 8b^2$; $4^0 = 1$ multiply by 4 zero times; $9^1 = 1 \times 9 = 9$; $3^2 = 1 \times 3 \times 3 = 9$
$a^{-n} = 1 \div a \div a \div a \ldots$ (n times) $= \frac{1}{a^n}$	negative integers → repeated division	$a^1 = 1 \times a = a$; $a^{-1} = 1 \div a = \frac{1}{a}$; $5^{-2} = 1 \div 5 \div 5 = \frac{1}{5^2} = \frac{1}{25} = \frac{4}{100} = 0.04$
$a^ma^n = a^{(m+n)}$; $\frac{a^m}{a^n} = a^{(m-n)}$	to multiply terms*, add exponents; to divide, subtract exponents *Must have same base.	$(a^2)(a^3) = a^{2+3} = a^5$; $(\sqrt{7})(\sqrt{7}) = 7^{\frac{1}{2}}7^{\frac{1}{2}} = 7^{\frac{1}{2}+\frac{1}{2}} = 7^1 = 7$; $\frac{2a^5}{2^4a^3} = \frac{a^{(5-3)}}{2^{(4-1)}} = \frac{a^2}{2^3} = \frac{a^2}{8}$
$(a^m)^n = a^{mn}$; $(ab)^n = a^nb^n$; $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	to raise a term to a power, multiply exponents; apply the power to all the term's constants and variables	$(g^2)^3 = g^{(2\cdot3)} = g^6$; $(2a^5)^3 = 2^{(1\cdot3)}a^{(5\cdot3)} = 2^3a^{15}$ [illegible]; $\left(\frac{2a^3}{b}\right)^2 = \frac{2^{(1\cdot2)}a^{(3\cdot2)}}{b^{(1\cdot2)}} = \frac{2^2}{}$ [illegible]
$a^{\frac{1}{n}} = \sqrt[n]{a}$; $a^{\frac{m}{n}} = \sqrt[n]{a^m}$	a value raised to power $\frac{1}{n}$ equals the "n^{th} root of" the value; $a^{\frac{1}{2}} = \sqrt{a}$ (square root)	$2^{\frac{3}{2}} = 2^{(3)\frac{1}{2}} = \sqrt{2^3} = \sqrt{}$ [illegible] $\sqrt{2}$; $\sqrt[3]{27} = 27^{\frac{1}{3}} = (3$ [illegible]; $\sqrt[4]{x^8} = \sqrt[4]{x^4x^2} =$ [illegible] $x\sqrt{x}$

Examples: Group numbers and common bases to simp[illegible]

$\sqrt{18} = \sqrt{(9)(2)} = 3\sqrt{2}$ $\quad \frac{16\sqrt{64}}{4^4} = \frac{(16)(8)}{4^4} = \frac{(4^2)(4)(2)}{4^4} = \frac{2}{4^{(4-3)}} = \frac{2}{4} = \frac{1}{2}$ $\quad \left(\frac{4x^{-2}y}{y^3}\right)^{}$ [illegible]

$(3x^2y)(4xy^3) = (3)(4)(x^2)(x)(y)(y^3) = 12x^{2+1}y^{1+3} = 12x^3y^4$ $\quad \left(\frac{3}{5a}\right)^{-2} = \frac{3^{-2}}{5^{-2}a^{-2}}$ [illegible] $= \left(\frac{2}{xy}\right)^3 = \frac{8}{x^3y^3}$

10:A BCD SIMPLIFYING POLYNOMIAL EXPRESSIONS

polynomial: expression with more than one term (degree = highest power of x) — Ex[illegible] 2^{nd} de[illegible]

Rule or Reminder	Simplifi[illegible] Example
add/subtract polynomials: like ter[illegible]	$3x^2 + x - 2$ [illegible]
distributive property: $a(b + c) =$ [illegible]	[illegible] $x + 8$
multiplying two polynomials: m[illegible] in the 1^{st} polynomial by each [illegible] polynomial; use a grid or, for two [illegible] (2 terms), use **FOIL: First, Outer, Inne**[illegible] *Recall $a - b = a + (-b)$ to avoid sign erro*[illegible]	[illegible] $+ 2x - 8$
dividing two polynomials, polynomial long division: 1. write terms from highest to lowest powers (missing powers need coefficient of 0) [illegible] only the first terms at each [illegible] each value of the [illegible] the dividend; [illegible] the whole polyn[illegible] the remai[illegible] if any, is expresse[illegible] divis[illegible]	[illegible]
PEMDAS order of o[illegible]: **P**arentheses, **E**xponents, **M**ultiply/**D**ivide left to rig[illegible], **A**dd/**S**ubtract left to right	[illegible] $(2x) + 7x = 2x$ [illegible] $1)^2 + 7x =$ [illegible] $+ 1)(-4x + 1) +$ [illegible] $^2 - 4x - 4x + 1)$ [illegible] $6x^2 - 8x + 1) +$ [illegible] $16x^2 + 2x$ [illegible] $32x$

[illegible] FACTORS AND DI[illegible] OF SQUARES

[illegible] **common factor, GCF:** la[illegible] value by which eac[illegible] a polynomi[illegible] divisible; r[illegible] se distributive prop[illegible]

Exam[illegible] **F**: $28x^3 - 8x^2$ [illegible] $\to 4x(7x^2 -$ [illegible]

diff[illegible] **square**[illegible] ^{2}: binom[illegible] at [illegible] $b)(a + b)$; [illegible] (FOIL) to show [illegible] $+ b) = a^2 + a$ [illegible] $b^2 =$ [illegible] ^{2}

Example: x[illegible] (difference [illegible] so $x^2 - 9 =$ [illegible]

Examples: Decide [illegible] each polynomial can be (1) fa[illegible] finding th[illegible] re[illegible] using the difference

$27x^2 -$ [illegible] $3[(3$ [illegible] $- (2)^2] = 3(3x - 2)(3x + 2)$

$144x^2$ [illegible] $(12x$ [illegible] $1)$ $\quad 144x^2 + 9 \to$ *not poss*[illegible]

$16x^4 - 81 \to (4x^2 - 9)(4x^2 + 9) = (2x - 3)(2x + 3)(4x^2 + 9)$

cancelling: cancel com[illegible] factors found in both [illegible] numerator and denominator

Example: Use cancelling: $(2x$ [illegible] $\quad \frac{2x^2 - 72}{2x + 12} = \frac{2(x^2 - 36)}{2(x + 6)} = \frac{(x - 6)(x + 6)}{(x + 6)} = x - 6$

10: E FACTORING TRINOMIALS

A **trinomial** (3 terms) in [illegible] $ax^2 + b$[illegible] can be "unFOILed" (fa[illegible] and rew[illegible] $(mx + p)(nx + q)$ if 3 [illegible] ca[illegible]

1. $mn = a$ **2.** m[illegible] $= b$ **3.** [illegible]

- Factor out any GCF fi[illegible] of x^2 is negative, factor out [illegible] — [illegible] pproach us[illegible] where [illegible] $c, m, n, p,$ [illegible] e integers
- For $x^2 + bx + c$, you know **1.** m[illegible] [illegible] therefore you only need to find p and q, such that [illegible] $pq =$ [illegible]

Example: Factor $3x^2 + 6x - 189$. → $3(x^2 +$ [illegible] $(x +$ [illegible] $)(x - \_\_)$
→ p and q must have opposite signs becaus[illegible] is negative
→ [illegible] sitive value must be +2 greater magnitude than negative value
→ [illegible] ctor pair of 63 {1 x 63; 3 x 21; [illegible]} is 2 apart: $3(x + 9)(x - 7)$
[illegible] guess: $(x + 9)(x - 7) =$ [illegible] $7x$ [illegible] $x^2 + 2x - 63$ ✓

- If a [illegible] be rewritten as [illegible]^2 or [illegible]$)^2$, then it is a **perfect s**[illegible]**l**: for $(m$[illegible] $x + p), mp + mp = 2mp = b$

Examples: Fa[illegible] [illegible] and 81 are squares of $2x$ and 9 [illegible] heck to see w[illegible] al is a perfect square, $(2x + 9)^2$: [illegible] $9)(2x + 9) = 4x$ [illegible] $x + 81 = 4x^2 + 36x + 81$ ✓
[illegible] $^2 + 6x - 9$. → $-(x^2$ [illegible] $+ 9) = -(x - \_\_)(x - \_\_)$
[illegible] must have same signs because 9 is positive; and those [illegible] must be negative signs [illegible] use -6 is negative ($p + q = -6$)
[illegible] actor pair of 9 {1 x 9; 3 [illegible]} sums to 6: $-(x - 3)(x - 3) = -(x - 3)^2$

[illegible] by thinking/guessing/checking, [illegible] sting all* possibilities in a table, or [illegible] sing the box method (see example). *If p_{guess} and q_{guess} result in $-b$, then the correct p and q values are $-p_{guess}$ and $-q_{guess}$.

[illegible] s: If possible, [illegible] or $6x^2 - 11x - 10$.
[illegible] m and n must be a factor pair of 6 {1 x 6 or 2 x 3}
[illegible] p and [illegible] must be a factor pair of −10 {−1 x 10; −2 x 5; or the [illegible] site [illegible] q values*}; find $m, n, p,$ and q such that $b = -11$

$(mx + p)$	$(nx + q)$	$mq + pn = b$
$(1x - 1)$	$(6$[illegible]	$10 - 6 = 4$
[illegible] $x + 10)$	$(6$[illegible]	$-1 + 60 = 59$
[illegible] $- 2)$	$(6x + 5)$	$5 - 12 = -7$
[illegible] $x + 5)$	[illegible] $- 2)$	$-2 + 30 = 28$
$(2x$ [illegible]	[illegible] $x + 10)$	$20 - 3 = 17$
$(2x + 1$[illegible]	$(3x - 1)$	$-2 + 30 = 28$
[illegible] $- 2)$	$(3x + 5)$	$10 - 6 = 4$
[illegible] $+ 5)$	$(3x - 2)$	$-4 + 15 = 11$*

[illegible] $= -11$, so $(2x - 5)(3x + 2)$
[illegible] $6x^2 + 4x - 15x - 10 = 6x^2 - 11x - 10$ ✓

Box Method Steps	Example
Put ax^2 in top left and c in bottom right of box.	$6x^2$ / -10
Multiply $a \cdot c$; find the product's two factors that also sum to b.	$6 \cdot (-10) = -60$; $4 \cdot (-15) = -60$; $4 + (-15) = -11$
Add two factors to box as coefficients of x.	$6x^2$, $-15x$; $4x$, -10
Find GCF of each row and column (or −GCF as needed). Write factored form: $(2x - 5)(3x + 2)$	$2x$, -5; $3x$: $6x^2$, $-15x$; $+2$: $4x$, -10

12: ABE RECOGNIZING, EVALUATING, AND SOLVING FUNCTIONS

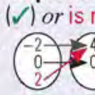
domain: set of all defined x-values

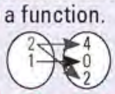
range: set of all defined y-values

function, $f(x)$: relation in which each x-value (input) has one y-value (output)

- has no repeated x-values
- passes vertical line test, VLT (cannot cross graph at more than one point)

Examples: Decide whether each is (✓) or is not (✗) a function.

1.  ✓ function

2.  ✗ ($x = 2$ repeats)

3. There are 4 cups, c, per box, b. → $c = 4b$; {(0,0); (1,4); (2,8); ...} is set of (b, c) ✓ function

4. [illegible] **5.** [illegible]

[illegible] 2 [illegible] 8 [illegible] eat) [illegible] nts) [illegible] ain. [illegible] s 0" [illegible] 10 [illegible] = −6 [illegible] ng it. [illegible] $a \times \frac{1}{2}$ [illegible] $\frac{1}{2}(9)$

acceleration, a, in m/s². For a 2 kg ball, solve for $a(F)$. What is $a(9)$? $= 4.5$ m/s² acceleration (caused by a 9 N force on a 2 kg ball)

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CR Symbol indicates section aligns to Texas College and Career Readiness Standards.